

Hong Kong Mathematics Olympiad (1990 – 91)

Heat Event (Individual)

香港数学竞赛 (1990 – 91)

初赛项目 (个人)

- Find the value of $\log_3 14 - \log_3 12 + \log_3 486 - \log_3 7$.
求 $\log_3 14 - \log_3 12 + \log_3 486 - \log_3 7$ 的值。
- A scientist found that the population of a bacteria culture doubled every hour. At 4:00 pm, he found that the number of bacteria was 3.2×10^8 . If the number of bacteria in that culture at noon on the same day was $N \times 10^7$, find N .
某科学家发现某样本中细菌的数量每小时增加一倍。于下午四时，他发现细菌的数量为 3.2×10^8 ，若于同日正午该样本中细菌的数量为 $N \times 10^7$ ，求 N 。
- If $x + \frac{1}{x} = 8$, find the value of $x^3 + \frac{1}{x^3}$.
若 $x + \frac{1}{x} = 8$ ，求 $x^3 + \frac{1}{x^3}$ 的值。
- If the equations $2x + 3y + a = 0$ and $bx - 2y + 1 = 0$ represent the same line, find the value of $6(a + b)$.
若方程 $2x + 3y + a = 0$ 及 $bx - 2y + 1 = 0$ 代表同一直线，求 $6(a + b)$ 的值。
- A boy walks from home to school at a speed of 2 metres per second and runs back at x metres per second. His average speed for the whole journey is $2\frac{2}{3}$ metres per second. Find x .
某童以每秒 2 米的速度由家步行回校，又以每秒 x 米的速度跑回家。若该童的往返平均速度为每秒 $2\frac{2}{3}$ 米，求 x 。
- The straight line $\frac{ax}{3} - \frac{2by}{5} = 2a + b$ passes through a fixed point P . Find the x -coordinate of P .
直线 $\frac{ax}{3} - \frac{2by}{5} = 2a + b$ 恒过一定点 P ，求 P 的 x 坐标。
- If the diameter of a sphere is increased by 20%, its volume will be increased by $x\%$. Find x .
若一球体的直径增加 20%，则其体积增加 $x\%$ ，求 x 。
- If $\log_7[\log_5(\log_3 x)] = 0$, find x .
若 $\log_7[\log_5(\log_3 x)] = 0$ ，求 x 。

9. If $\frac{7-8x}{(1-x)(2-x)} = \frac{A}{1-x} + \frac{B}{2-x}$ for all real numbers x where $x \neq 1$ and $x \neq 2$, find $A + B$.
 若 $\frac{7-8x}{(1-x)(2-x)} = \frac{A}{1-x} + \frac{B}{2-x}$, 其中 x 为实数, 且 $x \neq 1$ 及 $x \neq 2$, 求 $A + B$ 。
10. The marked price of an article is $p\%$ above its cost price. At a sale, the shopkeeper sells the article at 20% off the marked price. If he makes a profit of 20%, find p .
 某商品的标价比成本高出 $p\%$ 。在一次大减价中, 店主以「照价八折」的价钱售出该商品。若该店主仍可得利润 20%, 求 p 。
11. If $a < 0$ and $2^{2a+4} - 65 \times 2^a + 4 = 0$, find a .
 若 $a < 0$, 且 $2^{2a+4} - 65 \times 2^a + 4 = 0$, 求 a 。
12. If one root of the equation $(x^2 - 11x - 10) + k(x + 2) = 0$ is zero, find the other root.
 设方程 $(x^2 - 11x - 10) + k(x + 2) = 0$ 的其中一根为零, 求另一根。
13. $[x]$ denotes the greatest integer less than or equal to x . For example, $[6] = 6$, $[8.9] = 8$, etc. If $[\sqrt[4]{1}] + [\sqrt[4]{2}] + \cdots + [\sqrt[4]{n}] = n + 2$, find n .
 $[x]$ 是小于或等于 x 的最大整数。例如, $[6] = 6$, $[8.9] = 8$ 等。若 $[\sqrt[4]{1}] + [\sqrt[4]{2}] + \cdots + [\sqrt[4]{n}] = n + 2$, 求 n 。
14. a , b are two different real numbers such that $a^2 = 6a + 8$ and $b^2 = 6b + 8$. Find the value of $\left(\frac{4}{a}\right)^2 + \left(\frac{4}{b}\right)^2$.
 a 、 b 为两个不同之实数, 且 $a^2 = 6a + 8$ 及 $b^2 = 6b + 8$, 求 $\left(\frac{4}{a}\right)^2 + \left(\frac{4}{b}\right)^2$ 的值。
15. $3^{12} - 1$ is divisible by an integer which is greater than 70 and smaller than 80. Find the integer.
 $3^{12} - 1$ 可被一个大于 70 及小于 80 的整数所整除, 求该整数。

16. It is known that

$$2^3 - 1^3 = 3 \times 1^2 + 3 \times 1 + 1$$

$$3^3 - 2^3 = 3 \times 2^2 + 3 \times 2 + 1$$

$$4^3 - 3^3 = 3 \times 3^2 + 3 \times 3 + 1 \quad .$$

$$\vdots \quad \vdots$$

$$101^3 - 100^3 = 3 \times 100^2 + 3 \times 100 + 1$$

Find the value of $1^2 + 2^2 + 3^2 + \cdots + 100^2$.

已知

$$2^3 - 1^3 = 3 \times 1^2 + 3 \times 1 + 1$$

$$3^3 - 2^3 = 3 \times 2^2 + 3 \times 2 + 1$$

$$4^3 - 3^3 = 3 \times 3^2 + 3 \times 3 + 1$$

$$\vdots \quad \vdots$$

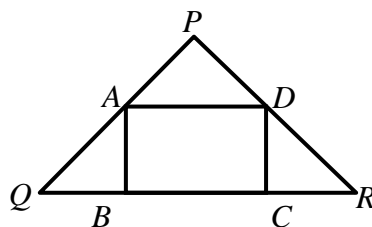
$$101^3 - 100^3 = 3 \times 100^2 + 3 \times 100 + 1$$

求 $1^2 + 2^2 + 3^2 + \cdots + 100^2$ 的值。

17. In figure 1, $PQ = PR = 8$ cm and $\angle QPR = 120^\circ$. A , D are the mid-points of PQ , PR respectively.

If $ABCD$ is a rectangle of area \sqrt{x} cm^2 , find x .

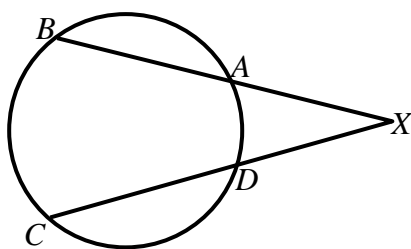
在图 1 中, $PQ = PR = 8$ cm 及 $\angle QPR = 120^\circ$. A 、 D 依次为 PQ 、 PR 的中点。若 $ABCD$ 是一个面积为 \sqrt{x} cm^2 的矩形, 求 x 。



(Figure 1)(图 1)

18. In figure 2, $XA = 10$ cm , $AB = 2$ cm , $XD = 8$ cm and $DC = x$ cm . Find the value of x .

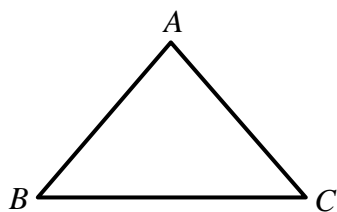
在图 2 中, $XA = 10$ cm、 $AB = 2$ cm、 $XD = 8$ cm 及 $DC = x$ cm, 求 x 的值。



(Figure 2)(图 2)

19. In figure 3, $AB = AC = 6$ cm and $BC = 9.6$ cm . If the diameter of the circumcircle of $\triangle ABC$ is x cm , find x .

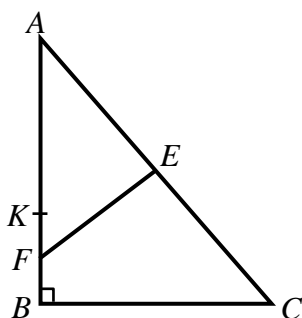
在图 3 中, $AB = AC = 6$ cm 及 $BC = 9.6$ cm。若 $\triangle ABC$ 的外接圆的直径是 x cm, 求 x 。



(Figure 3)(图 3)

20. In figure 4, $\angle ABC = 90^\circ$, $AK = BC$ and E , F are the mid-points of AC , KB respectively. If $\angle AFE = x^\circ$, find x .

在图 4 中, $\angle ABC = 90^\circ$ 、 $AK = BC$ 及 E 、 F 依次为 AC 、 KB 的中点。若 $\angle AFE = x^\circ$, 求 x 。



(Figure 4)(图 4)